A New Termination Condition for the Application of FDTD Techniques to Discontinuity Problems in Close Homogeneous Waveguide

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***Abstract*— It would be useful to avail a "general purpose" algorithm such as finite difference in the time domain to analyze discontinuity problems in classical waveguide. Its direct application, however, has been rendered difficult so far by the absence of exact termination conditions appropriate to the close waveguide environment. In the present contribution, a novel rigorous termination condition specific to homogeneous waveguide is introduced, that is based on the convolution proprieties of the modal characteristic impedance of the "accessible modes." This condition is straightforward to implement, as demonstrated by application to the nontrivial problem of a five cavity inductive post filter. Numerical results are compared to existing analytical and experimental data showing excellent accuracy.**

I. Introduction

It IS NOW well appreciated that direct numerical solution of Maxwell's equations by means of the FDTD algorithm, as proposed by Yee [1], offers the user a flexible analysis tool, requiring little analytical effort to the cost of some computer time. This is particularly important in situations where analytical methods may be lacking or tiresome to use, due to the complicated nature of the boundary conditions.

An important matter in the implementation of the method is the question of the proper termination of the field disturbance in space-time [2]. For a quasi-TEM-wave component (open region, boxed microstrip, etc.) Mur's conditions [3] effectively simulate absorption for near normal incidence on separation surfaces.

Their application to a close waveguide environment, where field incidence is anything but normal, however, yields reflections higher than 4% even 7%, if arrested to the first order. Recently, this condition has been applied individually to each plane wave component of a waveguide mode [4] and an alternative approximate condition, accurate around midband, has also been suggested [5].

Inability to simulate accurately a good match over the full waveguide band, in fact, remains today the major hindrance to the application of the FDTD method to classical waveguide problems. In order to overcome this difficulty, we employ the separability of the electromagnetic wave in close, homogeneous waveguide where mode patterns are frequency invariant. The proper termination of each mode, away from the disturbance, is expressed in the frequency domain by means of a characteristic impedance loading.

The Fourier transform of the latter from frequency-domain into time-domain is relatively simple. These considerations allow a matched termination in the time-domain to be expressed simply through the Fourier transforms of the characteristic impedances of the "accessible modes," i.e., the fundamental and the first few modes below cut-off, depending on the excitation and geometry, the latter either transport energy or cause near neighbour interaction between successive discontinuities.

This way, references planes can be set as close as desired to the actual discontinuities with considerable time-savings in the simulation.

This termination condition is tested by application to the nontrivial problem of a five cavity inductive post filter at Ka-band, that was analyzed earlier by analytical techniques [6]. Although results by the present method are in excellent agreement with the analytical ones and with experiment, it is not realistic to expect that the novel condition may enable FDTD to compete with one such analytical approach, where it exists. It is not difficult, however, to envisage situations where a more complex nature of the discontinuities may render the FDTD algorithm extremely useful to avail.

II. Analysis

Once the rectangular waveguide, including discontinuities, is discretized, in order to implement the FDTD algorithm it is necessary to identify six planes whereupon to impose the appropriate termination conditions. Four of these planes coincide with the metal walls of the guide, where tangential electric fields are made to vanish. The remaining two planes must be "absorbing surfaces" so as to simulate a field match as discussed next.

A second, shorter, empty auxiliary guide is also introduced in order to provide an excitation mechanism for the first. In this auxiliary guide, one of the nonmetal planes is taken as the plane of excitation, where the field pattern of the incident TE_{10} mode is imposed. The other plane is replaced by an ideal absorption condition, to be discussed later, simulating propagation in the infinite guide.

Essential to the following process is the ability to separate incident and reflected waves or "voltage" and "current" of each mode.

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In a close, homogeneous waveguide with perfectly conducting walls, the transverse fields at any given cross-section can be expressed in the complex frequency domain as

$$\tilde{\boldsymbol{E}}_t(\boldsymbol{r}, s) = \sum_n \tilde{V}_n(z, s)\boldsymbol{e}_n(x, y) \tag{1}$$

$$\tilde{\boldsymbol{H}}_t(\boldsymbol{r}, s) = \sum_n \tilde{I}_n(z, s)\hat{z} \times \boldsymbol{e}_n(x, y), \tag{2}$$

where the sum is over all TE and TM modes, $\boldsymbol{r} = (x, y, z)$, $s = \sigma + j\omega$ and $\boldsymbol{e}_n$ is a frequency independent field, orthonormalized over the guide cross-section.

The amplitudes $\tilde{V}, \tilde{I}$ in the frequency domain are given by

$$\tilde{V}_n(z, s) = \left\langle \tilde{\boldsymbol{E}}_t(\boldsymbol{r}, s), \boldsymbol{e}_n(x, y)\right\rangle$$

$$\tilde{I}_n(z, s) = \left\langle \tilde{\boldsymbol{H}}_t(\boldsymbol{r}, s) \times \hat{z}, \boldsymbol{e}_n(x, y)\right\rangle$$

($\langle \cdots \rangle$ denotes integration over the cross-section).

These equations imply separability of space and frequency dependences. Moreover, the "infinite guide" condition at any given cross-section z, omitting the now unnecessary z-dependence, is given by

$$\tilde{V}_n(s) = \tilde{Z}_{0n}(s)\tilde{I}_n(s), \tag{3}$$

where

$$\tilde{Z}_{0n}^{\mathrm{TE}}(s) = \frac{\eta s}{\sqrt{s^2 + \omega_n^2}} \tag{4a}$$

$$\tilde{Z}_{0n}^{\mathrm{TM}}(s) = \frac{\eta}{s}\sqrt{s^2 + \omega_n^2} \tag{4b}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon}}, \tag{4c}$$

and ω_n denotes the cut-off frequency of the nth mode.

By going over to the time-domain though the correspondence

$$\tilde{V}_n(s) \to V_n(t); \quad \tilde{I}_n(s) \to I_n(t); \quad \tilde{Z}_{0n}(s) \to Z_{0n}(t),$$

(3) becomes a convolution product denoted by $*$,

$$V_n(t) = Z_{0n}(t) * I_n(t). \tag{5}$$

We concentrate now on the TE case. Analogous expressions hold for the TM case.

We note that the following Laplace transform holds:

$$L[J_0(\omega_n t)] = \frac{1}{\sqrt{s^2 + \omega_n^2}}, \tag{6}$$

where J_0 is the Bessel function of zero order and $L\left[\frac{df}{dt}\right] = sF(s) - f(0)$. From (6), we obtain

$$L^{-1}\left[\frac{s}{\sqrt{s^2 + \omega_n^2}}\right] = \frac{d}{dt}J_0(\omega_n t) + \delta(t). \tag{7}$$

By substituting (7) into (5), we now recover for TE modes

$$V_n(t) = \eta\left[I_n(t) - \omega_n \int_0^t J_1(\omega_n \tau)I_n(t - \tau)d\tau\right]. \tag{8}$$

In practice, this convolution is effected by sampling the function $I_n(t)$ at the instants $(k - (1/2))\Delta t$ with k integer.

With a view to evaluating (8), we approximate $I_n(t)$ as accurately as desired by means of a staircase function of the type:

$$I_n(t) = \sum_{k=1}^{N} I_n\left[\left(k - \frac{1}{2}\right)\Delta t\right]U_0[(k-1)\Delta t, k\Delta t]$$
$$(N-1)\Delta t < t \leq N\Delta t, \tag{9}$$

where U_0 denotes the rectangular function of unit amplitude from $(k - 1)\Delta t$ to $k\Delta t$. When (9) is employed in (8), we obtain the expression:

$$V_n(N\Delta t) = \eta\left[I_n\left[\left(N - \frac{1}{2}\right)\Delta t\right] - \omega_n \sum_{k=1}^{N} I_n\left[\left(k - \frac{1}{2}\right)\Delta t\right] \int_{(N-k)\Delta t}^{(N-k+1)\Delta t} J_1(\omega_n t)dt\right] \tag{10}$$

that can be directly implemented on the computer. It is noted that the previous termination conditions need only be applied to the fundamental and the first few modes below cut-off, still causing appreciable interaction between successive discontinuities (accessible modes), normally, just the first mode below cut-off. Terminating as in (8) the accessible modes allows, in fact, the two transverse reference planes of the simulation to be chosen as close as desired at either side of each individual discontinuity in the structure under study. Once each discontinuity is characterized as a four port, say, ordinary network analysis can be applied to the whole cascade.

III. Results

The foregoing analysis was numerically tested for rectangular waveguide, where very accurate analytical and experimental results are available. The guide dimensions were chosen as a = 7.112 mm and b = 3.556 mm, corresponding to a band of operation from 26.5 to 40 GHz. The example considered, consisted of a five cavity inductive post filter with the following dimensions. Post width: 1.5 mm; post lengths: 0.408 mm, 4.948 mm, 3.395 mm, 3.398 mm, 2.932 mm, and 0.408 mm; cavity dimensions: 4.238 mm, 4.254 mm, 4.255 mm, 4.254 mm, and 4.238 mm.

In view of the fundamental mode excitation and of the symmetry of the discontinuities, TE_{n0} modes only are excited with field components E_y, H_x, and H_z and no variation along the y-direction, so that from (1)

$$E_y(x, y, z) = \sum_n V_n(z, t)e_n(x) \tag{11a}$$

$$-H_x(x, z, t) = \sum_n I_n(z, t)e_n(x) \tag{11b}$$

with

$$e_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}; \qquad \omega_n = \frac{n\pi}{a}c.$$

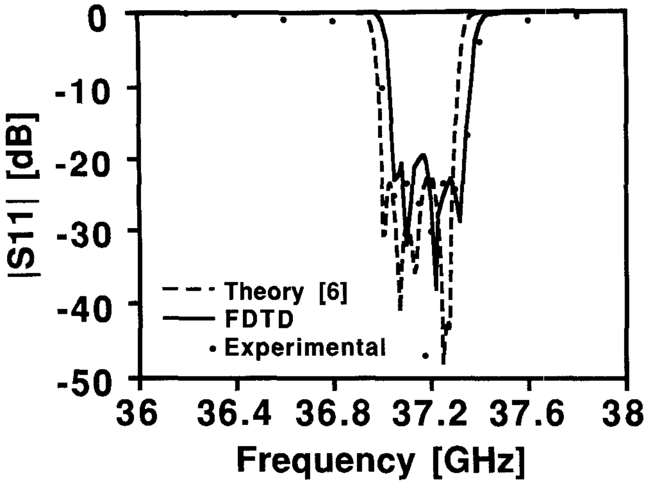

Fig. 1. Comparison of theoretical, experimental, and numerical FDTD results for S_{11} coefficient.

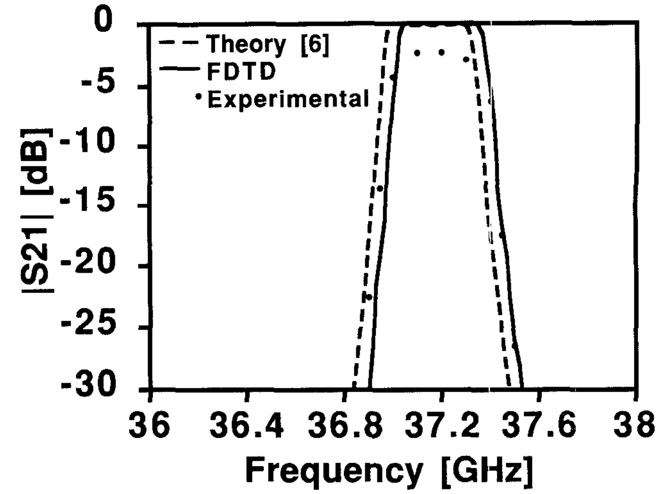

Fig. 2. Comparison of theoretical, experimental, and numerical FDTD results for S_{21} coefficient.

Variable discretization steps were taken so as to be able to fit accurately the geometry and whose length Δ was less than $\lambda_{\min}/20$, anyway.

The time-step Δt was chosen as 0.85 times the value dictated by the stability criterium found in [7], namely:

$$\Delta t \leq v_{\max}^{-1}\left[\frac{1}{\Delta x_{\min}^2}+\frac{1}{\Delta z_{\min}^2}\right]^{-\frac{1}{2}}.$$

Typical resulting numerical errors in the matching where about 1%. The excitation pulse is a sinusoidal electric field at f = 33.25 GHz, modulated by a gaussian distribution with 6.75 GHz bandwidth. The gaussian is fixed so that the amplitude of its transform is reduced to 5% of peak value at the band edges.

By the above choice of excitation, it is possible to work with a pulse centered in the band 26.5 $\div$ 40 GHz. Time sampling was effected over a span of 18 ns, so as to allow sufficient time decay of the fields.

Another important feature of the present simulation was the introduction of the edge-condition at the post edges, directly in the time-domain, as suggested in [8]. Its use greatly improves the accuracy and the running time of the simulation.

Finally, from the time behavior of the fields, it is possible to recover the S-parameters of the structure under test that were compared with the analytical results and experiment of [6] as shown in Figs. 1 and 2. It is noted that differences between these sets of results are of the same order as those produced by a mechanical tolerance of 10 μm, whereas not using the edge condition produces a shift of the resonant frequency of 80 $\div$ 100 MHz for the same grid size.

Numerical implementation was effected on a Cray U-MP 4/64 machine, as the present algorithm exploits very well its vectorial and parallel processing characteristics. 2-D execution required less than 300 seconds of CPU time, while a typical 3-D execution required about 1000 seconds CPU-time.

IV. Conclusion

We have introduced, for the first time to our best knowledge, a rigorous and effective termination condition for scattering problems in close homogeneous waveguide suitable for implementation in connection with a FDTD algorithm. Numerical results compare very well with existing analytical and experimental data for the nontrivial problem of a five cavity inductive post filter in rectangular waveguide at millimetric frequencies.

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